

magnitude more important than those arising out of the additional cross-terms when a [(3-by-3)+(6-by-6)] method is replaced by a 9-by-9 one. This is quite evident in the middle stages of refinement.

The author stands in obligation to many of his colleagues for the benefit of professional argument and to Burroughs Corporation for placing computing equipment at his disposal. Prof. Dr C. H. MacGillavry contributed the structure and the Weissenberg data of the triclinic centric monocalciumphosphite monohydrate as a guinea pig for our experiments. Ir Willem J. van de Lindt of Burroughs Professional Services was good enough to check over the computational detail of the relaxation procedures and to point out certain fast variants.

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The Crystal Structure of Cobalt Sulfate Hexahydrate*

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CoSO₄·6H₂O crystallizes in the monoclinic system, *C2/c*. The cell dimensions are

$$a = 10.032, b = 7.233, c = 24.261 \text{ \AA}; \beta = 98.37^\circ.$$

There are 8 molecules in the unit cell; the X-ray density is 2.006 g.cm.⁻³. The structure consists of sulfate tetrahedra and cobalt-centered water octahedra linked by a three dimensional network of hydrogen bonds. Eleven of the twelve hydrogen bonds are between water and sulfate oxygens; there is one water to water hydrogen bond. The average S-O sulfate distance is 1.46 Å; the average Co-O distance in the water octahedra is 2.11 Å; and the average hydrogen bond (O-H···O) is 2.8 Å. There are two crystallographically different Co⁺⁺ ions in the structure. The hydrogen atoms are assigned an ordered configuration which would not contribute to residual entropy at low temperatures.

Introduction

Calorimetric measurements by Rao & Giaouque (1960) showed some unaccountable residual entropy in crystals of CoSO₄·6H₂O at low temperatures. We have investigated the crystal structure in search of an explanation of the disorder. The resulting structure offers no possibility of disordered rings of hydrogen bonds such as were found in Na₂SO₄·10H₂O (Ruben, Templeton, Rosenstein & Olovsson, 1960), nor do we find any other explanation of the entropy discrepancy.

The crystal morphology was described by Marignac (1855), and Groth (1908).

Experimental

Crystals of CoSO₄·6H₂O were grown from a saturated solution at 50–55°. The intensity data were obtained by multiple film Weissenberg techniques. The film photography was done using Fe K α ($\lambda = 1.9373 \text{ \AA}$) X-rays, on Ilford Industrial G film. Intensities were estimated visually by comparison with a calibrated set of spots. The scaling factor used between multiple films of the same layer was $\exp(2.0 \text{ sec } \mu)$, where μ is the equi-inclination angle and 2.0 is an empirically determined constant from the data; this scaling factor varied from 7.4 at the zero layer ($\mu = 0^\circ$), to 12.9 at the sixteenth layer ($\mu = 38.7^\circ$).

The first single crystal of CoSO₄·6H₂O was enclosed in a 0.1 mm. glass capillary. At the time we felt the

* Work done under the auspices of the U.S. Atomic Energy Commission.

crystals would be unstable in the open atmosphere, however, later we found the crystals to be sufficiently stable to handle in air. This first crystal was aligned about the [101] direction. Seventeen layers were photographed and 737 reflections were observed of which 125 were below the detection limit and called zero. Later a second crystal was aligned about its b axis and photographed in air. An additional 99 zero layer reflections were measured of which 10 were below the detection limit. The sum total of intensities used was 836, of which 135 were recorded as zero.

For greater accuracy, the cell dimensions of $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$ were measured on a General Electric XRD-5 equipped with a goniostat using $\text{Mo } K\alpha$ ($\lambda = 0.7107 \text{ \AA}$) X-rays.

In order to solve the structure we found it necessary to take some reflection data from $\text{MgSeO}_4 \cdot 6\text{H}_2\text{O}$ and $\text{MgSO}_4 \cdot 6\text{H}_2\text{O}$ both of which are isomorphic with $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$.

A crystal of $\text{MgSO}_4 \cdot 6\text{H}_2\text{O}$ was grown from its saturated solution at $50\text{--}55^\circ$. It was aligned about its b axis and photographed with $\text{Fe } K\alpha$ X-rays with the Weissenberg technique. Intensities were recorded of 102 $h0l$ reflections of which 6 were below the detection limit. Cell dimensions were measured on the goniostat as with the cobalt isomorph.

$\text{MgSeO}_4 \cdot 6\text{H}_2\text{O}$ crystals were prepared at room temperature by allowing a filtered solution of dilute selenic acid and excess MgO to evaporate in an open dish. A crystal was aligned along its b axis and photographed with the Weissenberg technique with $\text{Fe } K\alpha$ X-rays. Intensities were recorded of 102 $h0l$ reflections of which 15 were below the detection limit. The cell dimensions were obtained from the Weissenberg films.

Cell dimensions and space group

The cell dimensions of $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$ are:

$$a = 10.032 \pm 0.004 \text{ \AA} \quad \beta = 98.37 \pm 0.03^\circ \\ (98.7^\circ \text{ by Marignac (1855)})$$

$$b = 7.233 \pm 0.003 \text{ \AA} \quad V = 1742 \pm 2 \text{ \AA}^3$$

$$c = 24.261 \pm 0.010 \text{ \AA} \quad Z = 8$$

Space group = $C2/c(C_{2h}^6)$

$$a:b:c = 1.387:1:3.354$$

(1.396:1:3.381 by Marignac (1855))

X-ray density = 2.006 g.cm.^{-3} : measured density = 2.000 g.cm.^{-3} (Goszner, 1907), 2.019 (Thorpe & Watt, 1880), 2.017 (*Handbook of Chemistry and Physics*, 1957).

The characteristic extinctions of the intensities indicated space group Cc or $C2/c$. As there are 8 molecules in the unit cell and $C2/c$ has an 8 fold general position as opposed to 4 for Cc , $C2/c$ appeared intuitively to be the most likely space group. A 'zero moment test' on the 3 dimensional data was computed (Howells, Phillips & Rogers, 1950) and the results indicated a center of symmetry, space group $C2/c$. The best evidence we have for the space group $C2/c$

is the successful solution of a chemically reasonable structure. The cell dimensions of $\text{MgSO}_4 \cdot 6\text{H}_2\text{O}$ are:

$$a = 10.110 \pm 0.005 \text{ \AA} \quad c = 24.41 \pm 0.01 \text{ \AA} \\ b = 7.212 \pm 0.004 \text{ \AA} \quad \beta = 98.30 \pm 0.04^\circ$$

The cell dimensions of $\text{MgSeO}_4 \cdot 6\text{H}_2\text{O}$ are:

$$a = 10.36 \pm 0.03 \text{ \AA} \quad c = 25.1 \pm 0.1 \text{ \AA} \\ b = 7.38 \pm 0.04 \text{ \AA} \quad \beta = 98.1 \pm 0.2^\circ$$

Determination of structure

The three-dimensional Patterson function of $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$ was computed. Attempts were made to associate the larger peaks with Co-Co, S-S, and Co-S vectors; however, two trial structures with Co atoms in general positions failed to give a reasonable Fourier projection in the b direction. Later it was learned that Co atoms are in two sets of special positions.

Crystals of the isomorphic compounds $\text{MgSO}_4 \cdot 6\text{H}_2\text{O}$ and $\text{MgSeO}_4 \cdot 6\text{H}_2\text{O}$ were prepared and photographed. From the Patterson projections calculated from the three sets of $h0l$ data we found the locations of the Co and S atoms.

Table 1. *The $h0l$ observed structure factors for $\text{MgSO}_4 \cdot 6\text{H}_2\text{O}$*

$h =$	-10	-8	-6	-4	-2	0	2	4	6	8	10
$l=0$	28	49	25	40	38						
2	29	39	0	49	46	0	53	40	17	30	30
4	37	24	39	34	45	22	64	9	20	34	
6	29	15	13	38	49	0	86	17	54	18	
8	0	35	40	42	64	34	7	14	45	44	
10		13	18	31	18	39	31	65	40	22	
12		0	33	46	16	46	98	64	40	18	
14		6	43	27	33	32	25	26	19		
16		38	15	11	11	65	24	13	40		
18		20	21	10	37	33	71	33	34		
20			0	51	39	46	32	49			
22				14	13	17	9	39			
24					12	38	41				

Table 2. *The $h0l$ observed structure factors for $\text{MgSeO}_4 \cdot 6\text{H}_2\text{O}$*

$h =$	-10	-8	-6	-4	-2	0	2	4	6	8	10
$l=0$	0	47	17	51	25						
2	20	24	33	67	80	0	83	24	37	0	16
4	20	12	13	50	41	33	56	31	7	27	
6	21	0	20	20	73	0	114	11	49	8	
8	0	38	34	0	71	42	10	26	17	24	
10	11	10	13	20	39	17	15	38	0	13	
12		16	16	59	0	50	55	48	11	7	
14		0	41	7	45	9	39	17	20		
16		23	0	10	0	58	15	12	21		
18		10	17	10	33	20	59	16	26		
20			0	32	15	33	15	34			
22				15	0	18	0	23			
24						13	23				

A set of trial oxygen positions was obtained from an $h0l$ Fourier projection of the $\text{MgSO}_4 \cdot 6\text{H}_2\text{O}$. The signs of 54 of the 96 non-zero structure factors were estimated from the location of the Mg and S positions.

Table 4. Atomic parameters for $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$

	x	y	z	B
Co_1	0	0	0	3.3 Å ²
Co_2	0	0.949 ± 0.001	$\frac{1}{2}$	3.0
S	0.868 ± 0.001	0.452 ± 0.001	0.1244 ± 0.0003	2.2
O_1	0.778 ± 0.001	0.601 ± 0.002	0.1352 ± 0.0006	2.6
O_2	Sulfate { 0.980 ± 0.002	0.446 ± 0.003	0.1695 ± 0.0007	3.9
O_3	oxygens { 0.918 ± 0.002	0.492 ± 0.003	0.0698 ± 0.0007	3.9
O_4	{ 0.799 ± 0.001	0.276 ± 0.002	0.1193 ± 0.0006	2.9
O_5	Water { 0.592 ± 0.001	0.723 ± 0.002	0.0469 ± 0.0006	2.8
O_6	octahedra { 0.535 ± 0.002	0.325 ± 0.002	0.0673 ± 0.0006	3.2
O_7	about Co_1 { 0.305 ± 0.002	0.557 ± 0.003	0.0216 ± 0.0006	3.3
O_8	Water { 0.885 ± 0.002	0.159 ± 0.002	0.2823 ± 0.0007	2.3
O_9	octahedra { 0.886 ± 0.002	0.740 ± 0.002	0.2832 ± 0.0007	2.7
O_{10}	about Co_2 { 0.858 ± 0.001	0.949 ± 0.003	0.1771 ± 0.0006	3.3

Table 5. Interatomic distances in $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$

Standard deviations are about ± 0.03 Å

	Sulfate oxygens				Water oxygens					
	O_1	O_2	O_3	O_4	about Co_1			about Co_2		
					O_5	O_6	O_7	O_8	O_9	O_{10}
Co_1					{ 2.11 2.11	{ 2.05 2.05	{ 2.14 2.14			
Co_2								{ 2.13 2.13	{ 2.12 2.12	{ 2.11 2.11
S	1.46*	1.44*	1.51*	1.45*						
O_1		2.36	2.40	2.40	2.77†					
O_2	2.36		2.43	2.37				2.79†		2.79†
O_3	2.40	2.43		2.40	2.72†	2.68†	2.93†			
O_4	2.40	2.37	2.40			2.78†	2.86†			2.76†
O_5	2.77†		2.72†			{ 2.89 2.99	{ 2.90 3.10			
O_6			2.68†	2.78†	{ 2.89 2.99		{ 2.94 2.99			
O_7			2.93†	2.86†	{ 2.90 3.10	{ 2.94 2.99				
O_8	2.79†	2.65†						2.97	{ 3.00† 3.03	{ 2.94 3.04
O_9		2.69†						{ 3.00† 3.03	2.99	{ 2.96 3.03
O_{10}	2.79†			2.76†				{ 2.94 3.04	{ 2.96 3.03	

* S-O bond in SO_4^{--} .

† Hydrogen bond distances.

The sulfur atoms and ten sets of oxygen atoms occupy the general $8(f)$ positions:

$$8(f) \quad x, y, z; \quad -x, -y, -z; \quad -x, y, \frac{1}{2}-z; \\ x, -y, \frac{1}{2}+z+C \text{ centering.}$$

A Fourier projection of the $h0l$ data of $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$ was calculated and the x and z parameters of the cobalt, sulfur and oxygen positions were evaluated. The y parameters were estimated from expected S-O bond distances in the sulfate ion and O-O distances about the hydrated cobalt ion.

Least squares refinement utilizing the full matrix (Busing & Levy, 1959) was performed for a total of 13 cycles on the IBM 704 computer. Atomic form factors of Co^{++} (Watson & Freeman, 1961), neutral S (Tomii & Stam, 1958), and neutral O (Hartree & Hartree, 1939) were used in these calculations. 836

intensity data were used, 737 of which were obtained from a crystal rotated about the [101], and 99 of which were obtained from the zero layer of a [010] rotation. 18 scale factors, 34 positional parameters and 13 isotropic temperature factors for a total of 65 parameters were varied. The overall R factor, where $R = \frac{\sum ||F_o| - |F_c||}{\sum |F_o|}$, was 54% before refinement commenced, dropped to 42% after the first refinement and then dropped steadily to the final value of 13%. The necessity to run so many cycles was in part due to certain blunders in the original estimate of the y parameters for the oxygen atoms. The final R factors are as follows:

$$R \text{ (including zeros)} = 13.0\% .$$

$$R \text{ (omitting zeros)} = 10.9\% .$$

The data from the [101] setting are shown in Table 3.

The atomic parameters are shown in Table 4. By an oversight the dispersion correction $\Delta f'$ of about -1.8 electrons (for Fe $K\alpha$) was omitted from the cobalt form factor. As a result, the temperature factors for cobalt in Table 4 are larger than the true values by an unknown amount.

Discussion of the structure and hydrogen bonding

The structure consists of discrete SO_4^{--} tetrahedra and $\text{Co}(\text{H}_2\text{O})_6^{++}$ octahedra, Fig. 1. There are just enough water molecules to satisfy this octahedral hydration of each cobalt ion.

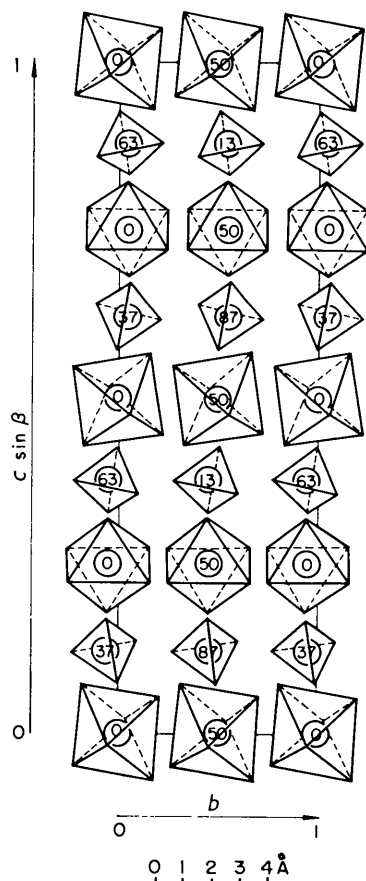


Fig. 1. Packing of SO_4^{--} tetrahedra and $\text{Co}(\text{H}_2\text{O})_6^{++}$ octahedra. The encircled numbers are the parameters of the Co and S atoms in the a direction.

Hydrogen bonds are assigned to the twelve shortest oxygen-oxygen distances (Table 5) after disregarding these between oxygen atoms of the same sulfate ion or of the same coordination octahedron. This assignment is checked by consideration of the hydrogen atom configuration and bond angles. Except for one water molecule (O_9) each water has two hydrogen bonds to sulfate oxygen atoms. Water molecule O_9 has one such bond and also a bond to atom O_8 . In

this bond geometry, there is only one configuration of hydrogen atoms with two hydrogen atoms on each water molecule. The angles between hydrogen bonds at these water molecules (Table 6) range from 97° to 126° and are within the range found for other hydrated sulfate crystals. The hydrogen bonds make angles of 111° to 124° with the water-cobalt vectors. The third hydrogen bond to O_8 makes angles of 80° and 117° with the two hydrogen bonds for which O_8 provides the hydrogen atoms.

Table 6. *Hydrogen bond angles*

O_1 through O_4 are sulfate oxygens
 O_5 through O_{10} are water molecules

Atoms	Angles
$\text{O}_1-\text{O}_5-\text{O}_3$	117°
$\text{O}_3-\text{O}_6-\text{O}_4$	105
$\text{O}_3-\text{O}_7-\text{O}_4$	126
$\text{O}_1-\text{O}_8-\text{O}_2$	97
$\text{O}_2-\text{O}_9-\text{O}_8$	116
$\text{O}_1-\text{O}_{10}-\text{O}_4$	124

The hydrogen bond lengths are 2.65 to 2.93 Å for the bonds to sulfate and 3.00 Å for the water-water

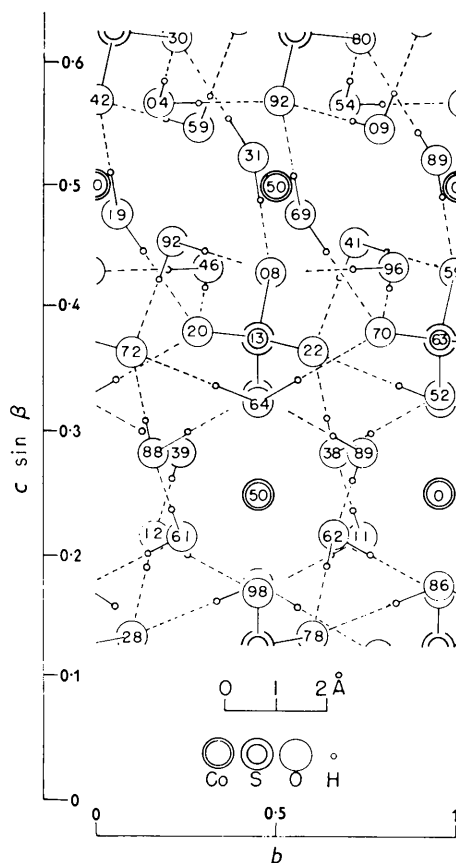


Fig. 2. Hydrogen bond network in $\text{CoSO}_4 \cdot 6\text{H}_2\text{O}$. The numbers are the parameters along the a axis of the Co, S, and O atoms. The complete environment about each of the two different $\text{Co}(\text{H}_2\text{O})_6^{++}$ and SO_4^{--} is shown.

bond. The shortest non-bonded oxygen-oxygen distances other than edges of the octahedra, are 3.13 Å between O₇ pairs (through a center of symmetry) and 3.22 Å between O₉ and O₄.

We find no reasonable alternative to this hydrogen configuration, and the entropy discrepancy is not explained by our work.

The hydrogen bonds tie the sulfate ions and water octahedra together in a three-dimensional network as indicated in Fig. 2. In this figure, the hydrogen configuration is indicated by small circles on the hydrogen bonds. Because of the bond angles the hydrogen atoms are expected to fall slightly off these lines joining oxygen atoms. We do not have a direct determination of the hydrogen positions from the diffraction data.

The two cobalt ions, Co₁ and Co₂, have point symmetries $\bar{1}$ and 2 respectively. The water octahedron about Co₁ is hydrogen bonded exclusively to sulfate ions. The Co₂ water octahedron has ten hydrogen bonds to sulfate ions and four involving water molecules of neighboring Co₂-type octahedra.

The average interatomic distances are as follows:

S-O(SO ₄ ²⁻)	1.46 Å
O-O(SO ₄ ²⁻)	2.39 Å
Co-O(Co(H ₂ O) ₆ ²⁺)	2.11 Å
O-H-O (hydrogen bond)	2.79 Å

Several other substances have the same structure as CoSO₄.6H₂O. In addition to the two magnesium compounds, CoSeO₄.6H₂O, ZnSO₄.6H₂O, and one form of NiSO₄.6H₂O are isomorphous (Groth, 1908). This structure has been recognized in nature as the

minerals hexahydrate, MgSO₄.6H₂O, and bianchite, ZnSO₄.6H₂O (Palache, Berman & Frondel, 1952).

A more thorough study of the structure of MgSO₄.6H₂O is in progress, and the location and refinement of the hydrogen parameters in that crystal will be the topic of another article shortly.

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The Crystal Structure of Ce₂₄Co₁₁*

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The structure of Ce₂₄Co₁₁, the most cerium-rich compound in the Ce-Co system, has been determined by single crystal X-ray methods. The compound is hexagonal, space group *P6₃mc* with *a* = 9.587 and *c* = 21.825 Å, *Z* = 2. The structure was solved by applying Buerger's minimum function. Nearly all of the Ce atoms have rather close Co neighbors, some Ce-Co distances being as short as 2.61 Å.

Introduction

Vogel (1947), in a study of the cerium-cobalt phase diagram, reported Ce₃Co as the most cerium rich compound in the system. Coffinberry (1960) prepared

alloys of this composition but was unable to obtain a single phase, excess cerium always being present. In fact, alloys containing as much as 30 at.% cobalt contained a small amount of excess cerium. It was not possible to establish the exact formula of this cerium rich compound by using metallographic techniques although it was known that the composition was approximately 30 at.% cobalt.

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